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MAXIMAL GEOIDAL ELEVATIONS DUE TO SIMULATED SEAMOUNTS. (U)  
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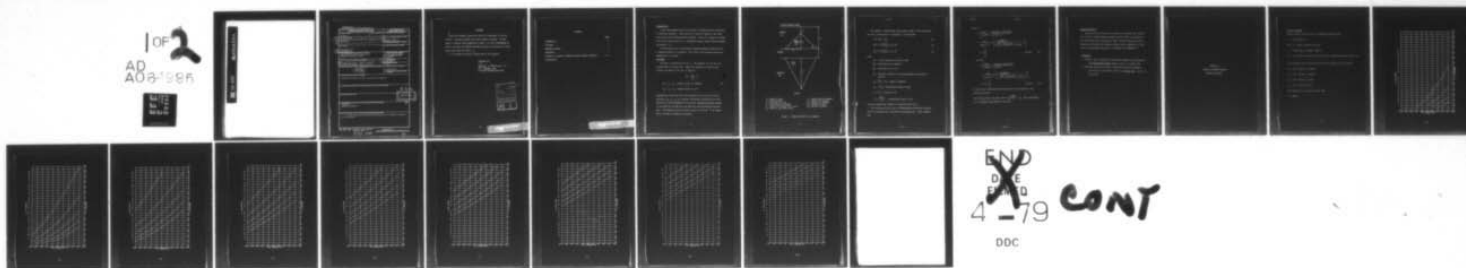
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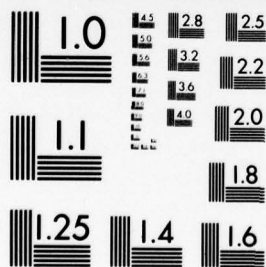
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>In this report maximal geoidal elevations due to simulated seamounts are calculated. |  |  |

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FOREWORD

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Satellite altimetry yields data about the topography of the sea surface. Seamounts beneath the surface affect its shape. In this report a seamount model, suggested to us by T. M. Davis (NAVOCEANO), is used to calculate the maximal geoidal elevation as a function of ocean depth, peak depth and slope. \*

A. R. DiDonato has kindly checked some of the formulas.

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## INTRODUCTION

It has been suggested that the presence of seamounts may be detectable by satellite altimetry. This would be of value in regions of the ocean that have not been bathymetrically surveyed. There are a number of studies that confirm the existence of a correlation between ocean bottom topography and gravity 1,2 .

In this study we are calculating the maximal geoidal elevation due to a simplified model of a seamount. The values of the densities have been suggested by T. M. Davis.

## THE MODEL

The model is illustrated in Fig. 1. The seamount and its root are represented by circular cones. Under the assumption of perfect Airy isostasy the depth of the root is given by

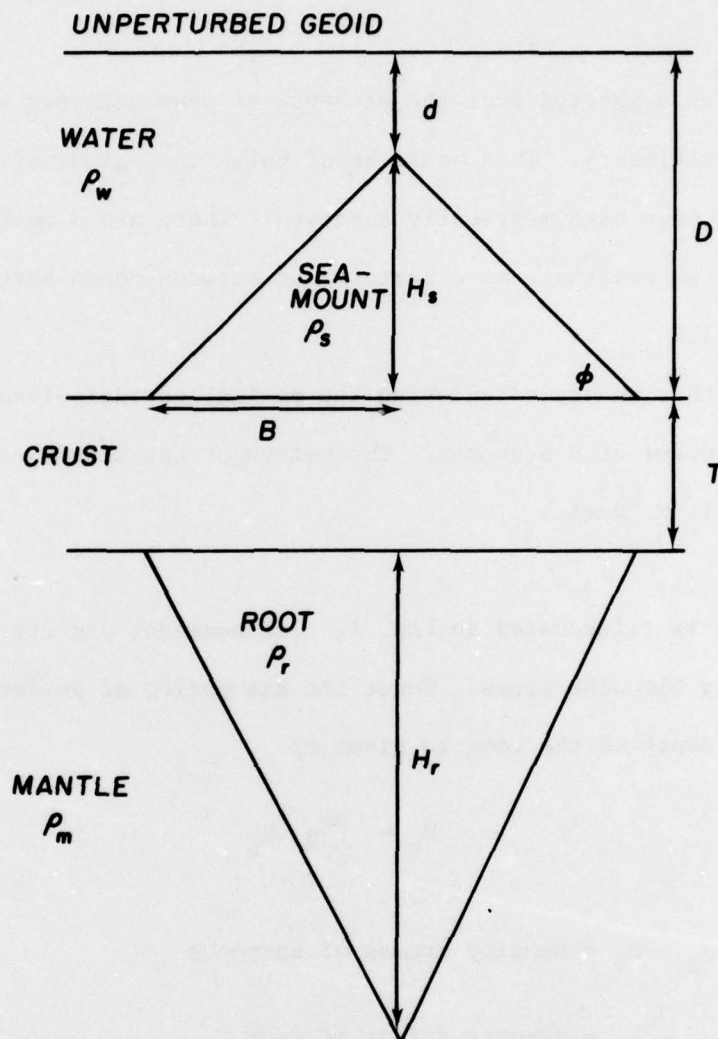
$$H_r = \frac{\Delta\rho_s}{\Delta\rho_r} H_s$$

$$\Delta\rho_s = \rho_s - \rho_w = \text{density excess of seamount} \quad (1)$$

$$\Delta\rho_r = \rho_m - \rho_r = \text{density defect of root}$$

Assuming the ocean depth (D), the thickness of the crust (T) and the densities ( $\rho_s$ ,  $\rho_w$ ,  $\rho_r$ ,  $\rho_m$ ) as given, the geoidal elevation due to the seamount (in the uncompensated or perfectly compensated models) depends on the depth of the peak (d), the slope ( $\phi$ ) and the distance from the axis. The maximal geoidal elevation occurs on the axis. It is expressible in terms of elementary functions.





#### LEGEND

- |  |                                  |
|--|----------------------------------|
| $D$ = depth of ocean                   | $\phi$ = slope angle of seamount |
| $d$ = depth of peak                    | $\rho_w$ = density of seawater   |
| $T$ = thickness of crust               | $\rho_s$ = density of seamount   |
| $B$ = base radius of seamount          | $\rho_r$ = density of root       |
| $H_s$ = height of seamount above crust | $\rho_m$ = density of mantle     |
| $H_r$ = depth of root below crust      |                                  |

Figure 1. Simplified Model of a Seamount

The seamount, constituting a mass excess, tends to lift the geoid; the root, a mass defect, to depress it. We may write

$$\Delta N = \Delta N_s - \Delta N_r \quad (2)$$

$$\Delta N_s = \Delta \rho_s H_s^2 \frac{G}{g} F_U (\alpha_s, \beta_s) \quad (3)$$

$$\Delta N_r = \Delta \rho_r H_r^2 \frac{G}{g} F_I (\alpha_r, \beta_r) \quad (4)$$

where

$\Delta N$  = total elevation of geoid on axis

$\Delta N_s$  = elevation due to seamount

$\Delta N_r$  = depression due to root

$\frac{G}{g}$  = universal constant of gravitation/mean acceleration of gravity

$\alpha_s = H_s/B = \text{tg} \phi$  = grade of seamount

$\beta_s = d/H_s$  = dimensionless depth of peak

$\alpha_r = H_r/B$  = grade of root

$\beta_r = \frac{D+T+H_r}{H_r}$  = dimensionless depth of root

The other symbols are defined in connection with Fig. 1.

The functions  $F_U$  and  $F_I$  are the dimensionless potentials along the axis of an upright and an inverted cone respectively. Their formulas are:

$$F_U(\alpha, \beta)$$

$$= 2\pi \left\{ -\frac{\alpha^2 \beta^2}{2(1+\alpha^2)} + \frac{1+\alpha^2(1+\beta)}{2\alpha(1+\alpha^2)} \sqrt{1+\alpha^2(1+\beta)^2} \right. \\ \left. + \frac{\alpha \beta^2}{2(1+\alpha^2)^{3/2}} \ln \left[ \frac{\alpha \beta (\sqrt{1+\alpha^2} - \alpha)}{\sqrt{1+\alpha^2} \sqrt{1+\alpha^2(1+\beta)^2} - 1 - \alpha^2(1+\beta)} \right] \right. \\ \left. - \beta - \frac{1}{2} \right\} \quad \text{for } \beta \geq 0 \quad (5)$$

$$F_I(\alpha, \beta)$$

$$= 2\pi \left\{ \frac{\alpha^2 \beta^2}{2(1+\alpha^2)} + \frac{1-\alpha^2(\beta-1)}{2\alpha(1+\alpha^2)} \sqrt{1+\alpha^2(\beta-1)^2} \right. \\ \left. + \frac{\alpha \beta^2}{2(1+\alpha^2)^{3/2}} \ln \left[ \frac{\alpha \beta (\sqrt{1+\alpha^2} + \alpha)}{\sqrt{1+\alpha^2} \sqrt{1+\alpha^2(\beta-1)^2} - 1 + \alpha^2(\beta-1)} \right] \right. \\ \left. - \beta + \frac{1}{2} \right\} \quad \text{for } \beta \geq 1 \quad (6)$$

No sphericity corrections have been made in the calculation of the geoidal elevations.

The limiting value of  $F_U$  when  $\beta \rightarrow 0+$  is  $\pi \left\{ \frac{\sqrt{1+\alpha^2}}{\alpha} - 1 \right\}$ . This corresponds to a vanishingly small depth of the peak.



### NUMERICAL RESULTS

Using (3) and (4) we have for different ocean depths (D), plotted the curves of constant maximal geoidal elevation (DN) in the plane of the slope (PHI) and the peak depth (DEL). Perfect isostasy has been assumed and therefore the computed quantity may be regarded as a lower limit of the maximal elevation to be expected (see Appendix A).

### REFERENCES

1. Fischer, Irene, "Deflections and Geoidal Heights Across Seamounts", Int. Hydrographic Review, Monaco, LIII (1), January 1976.
2. McKenzie, Dan and Carl Bowin, "Relationship Between Bathymetry and Gravity in the Atlantic Ocean", J. Geophy. Res., 81, No. 11, April 1976.



APPENDIX A  
GRAPHS OF CONSTANT MAXIMAL  
GEOIDAL ELEVATION

LEGEND FOR GRAPHS

Maximal geoidal elevations due to compensated seamount model.

$D$  = Ocean depth (m)

$DEL = d$  = Depth of seamount peak (m)

$\phi$  = Slope angle of seamount (degree)

$DN = \Delta N$  = Geoidal elevation on the axis due to compensated seamount model (cm)

The following values have been assumed for the densities (all in  $\text{gm/cm}^3$ ):

$\rho_s = 2.60$  = density of seamount

$\rho_w = 1.03$  = density of seawater

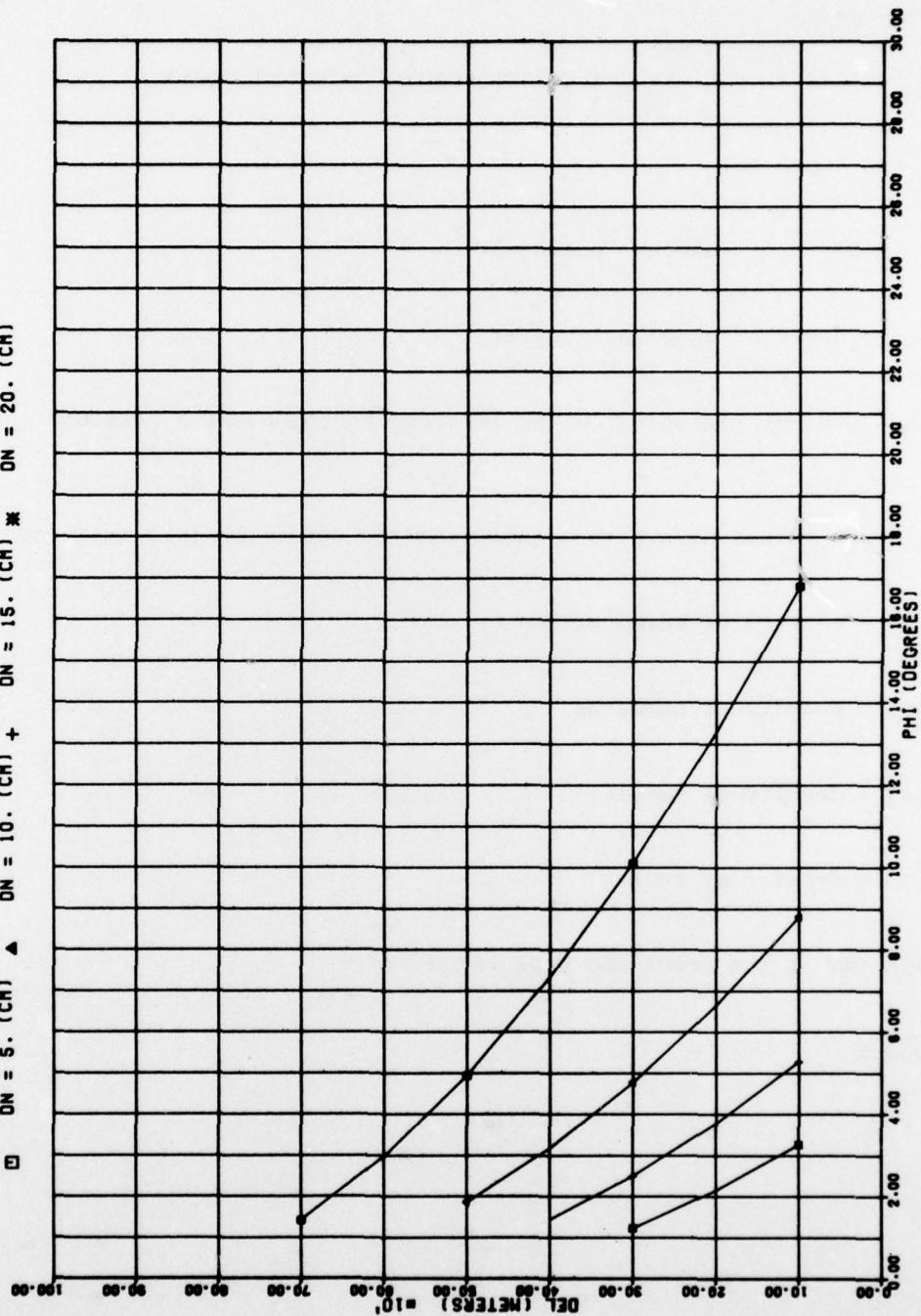
$\rho_m = 3.40$  = density of mantle

$\rho_r = 2.95$  = density of root

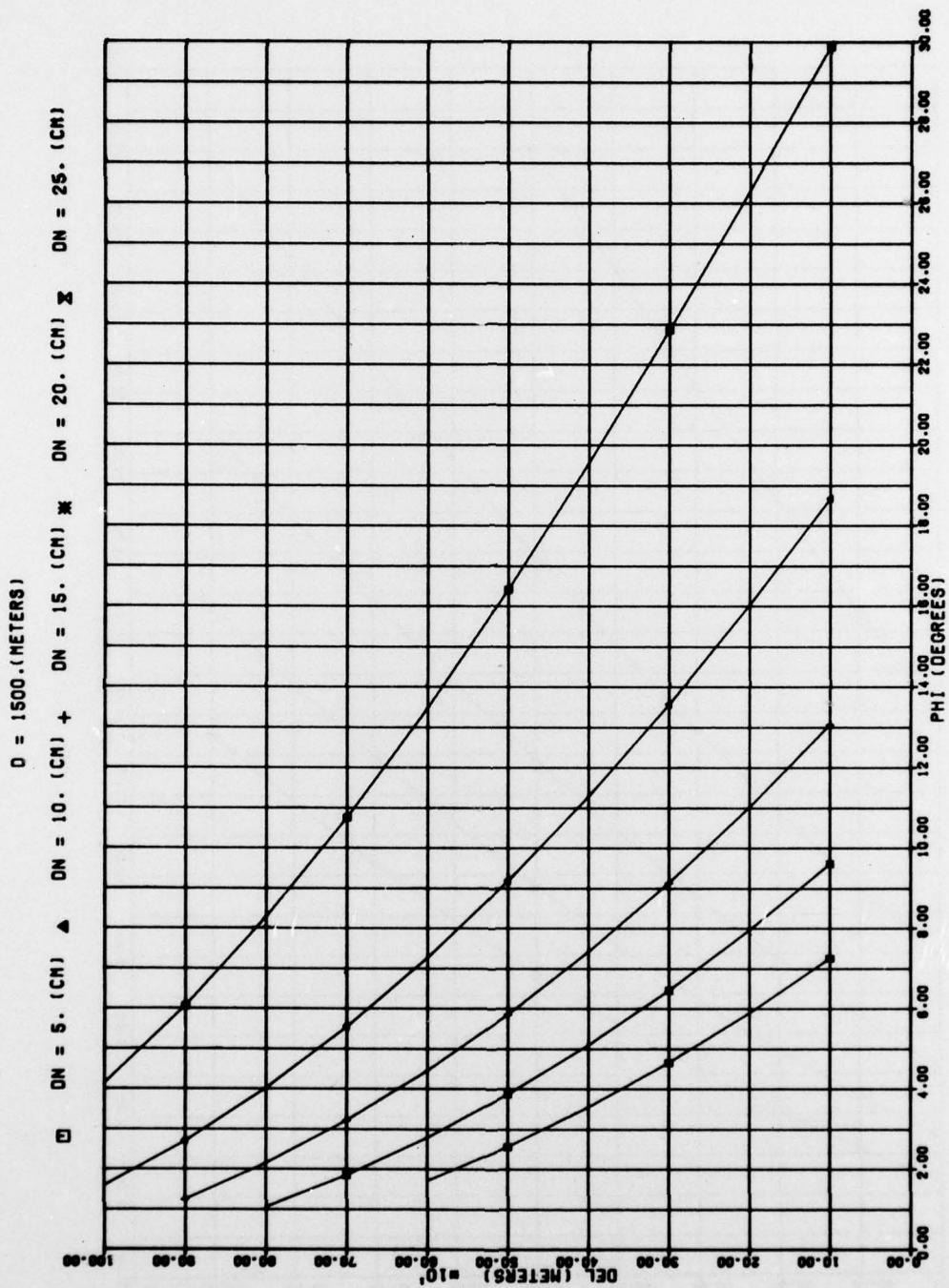
The thickness of the crust has been taken

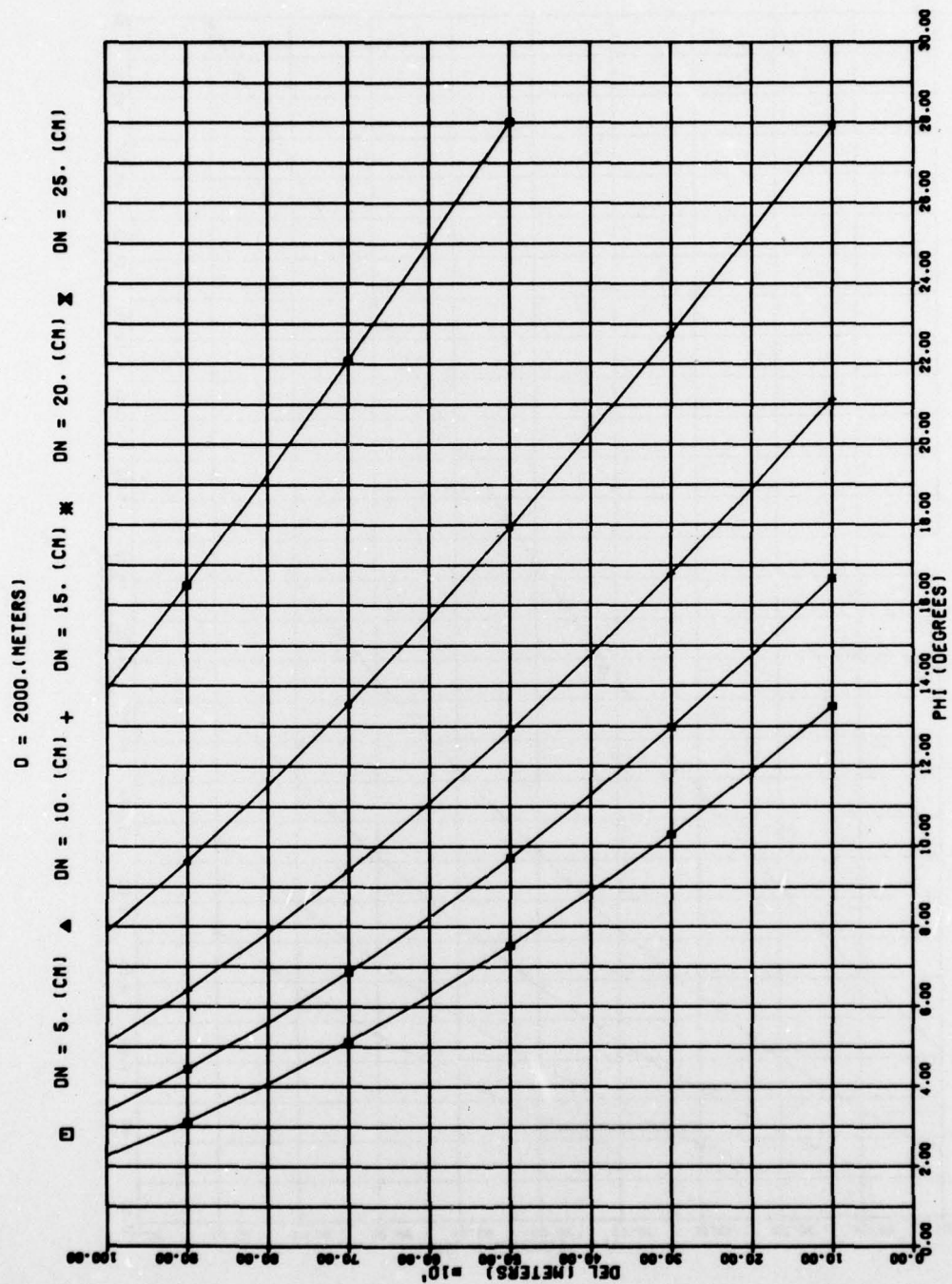
$T = 5000$  m

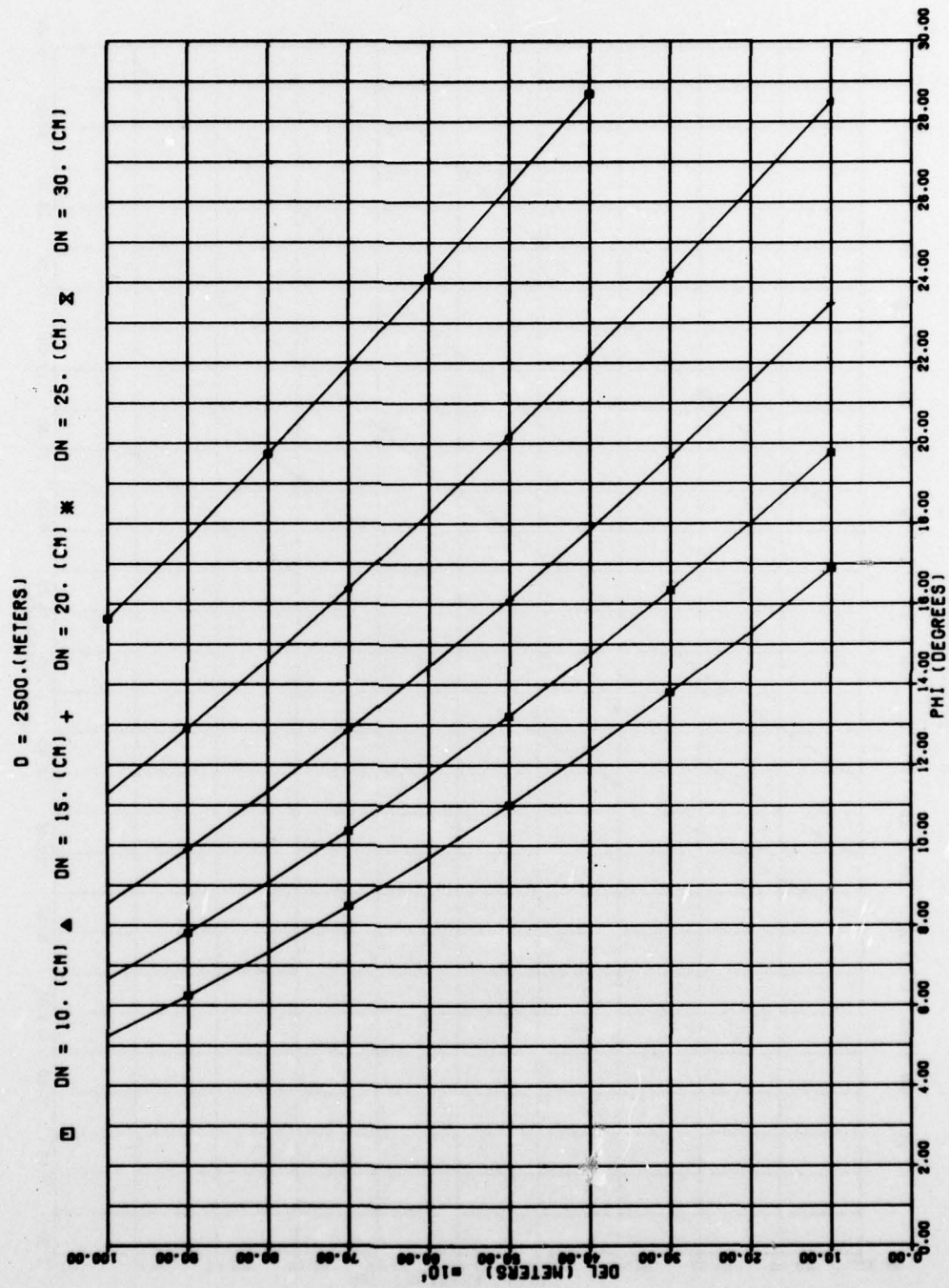
$D = 1000. (\text{METERS})$   
 $\square$   $DN = 5. (\text{CM})$     $\triangle$   $DN = 10. (\text{CM})$     $\times$   $DN = 15. (\text{CM})$     $\circ$   $DN = 20. (\text{CM})$



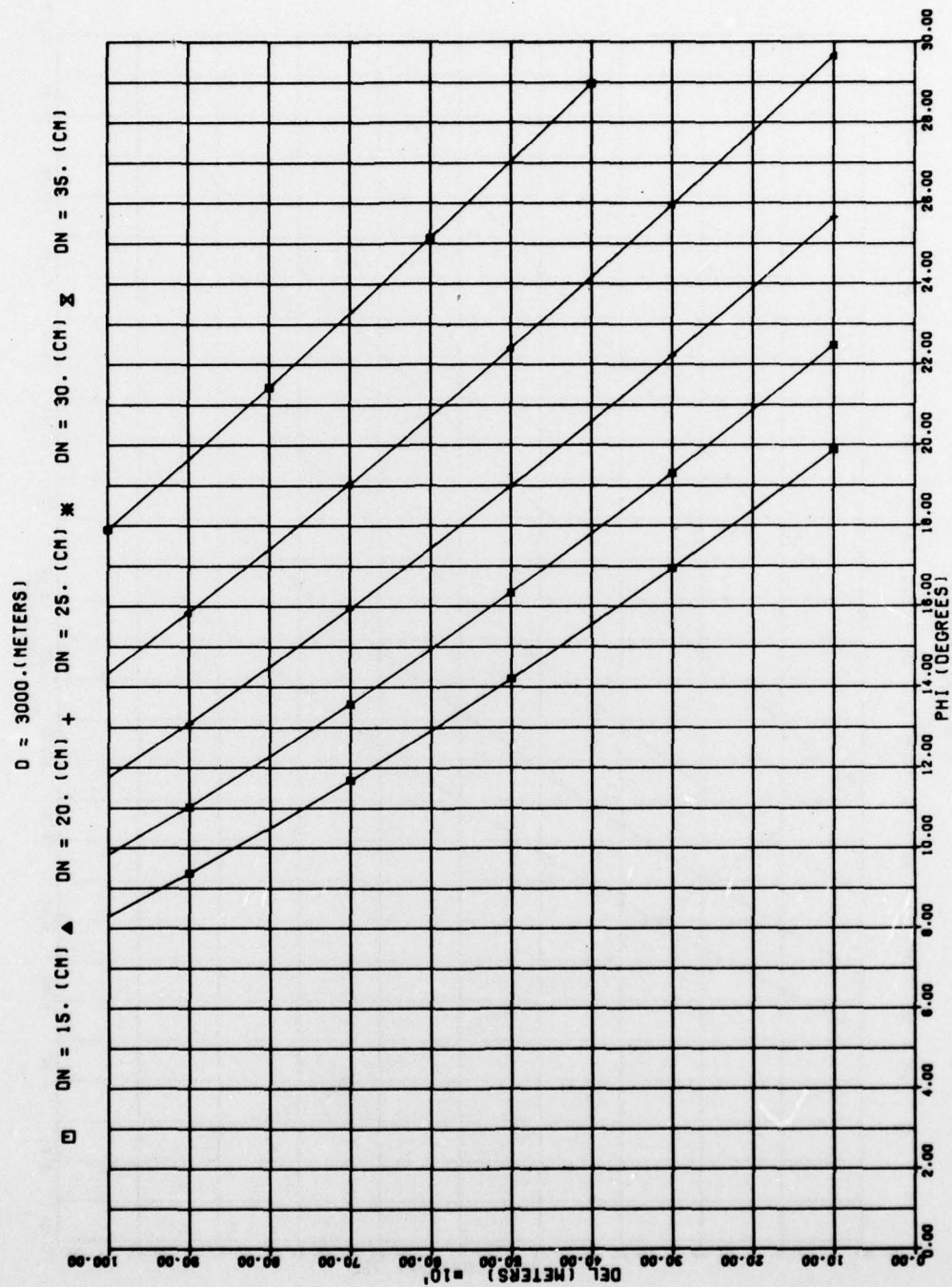




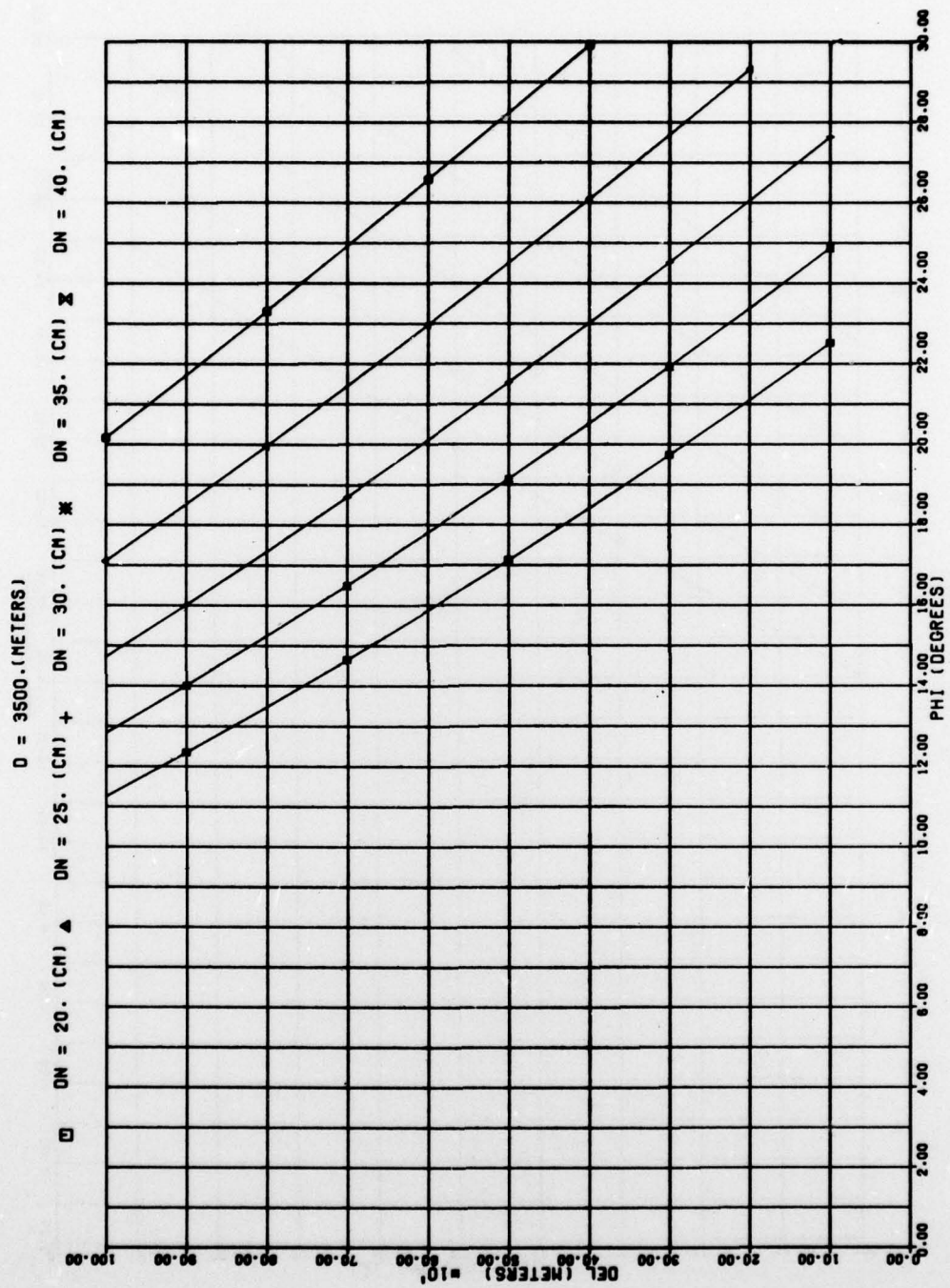


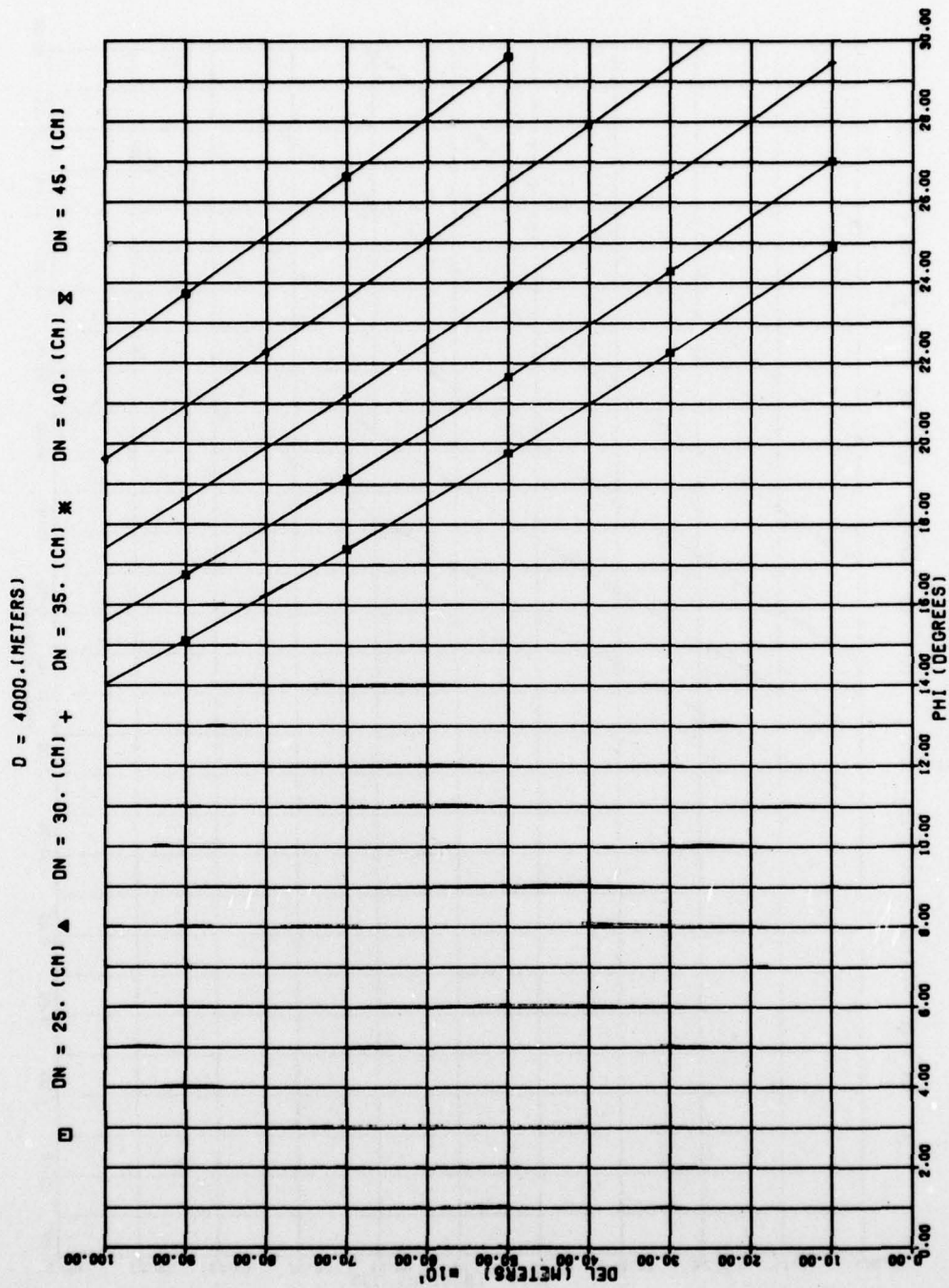


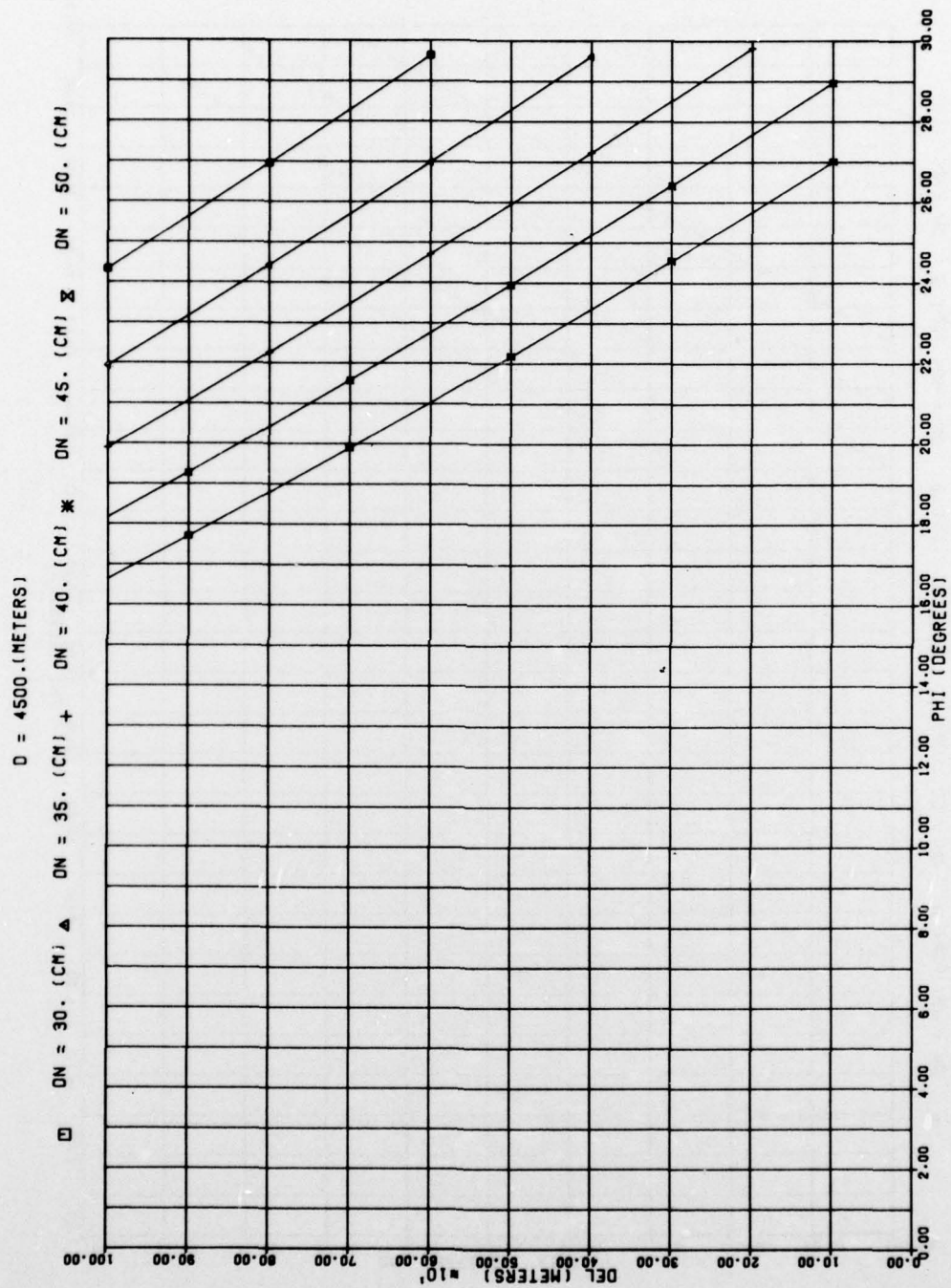




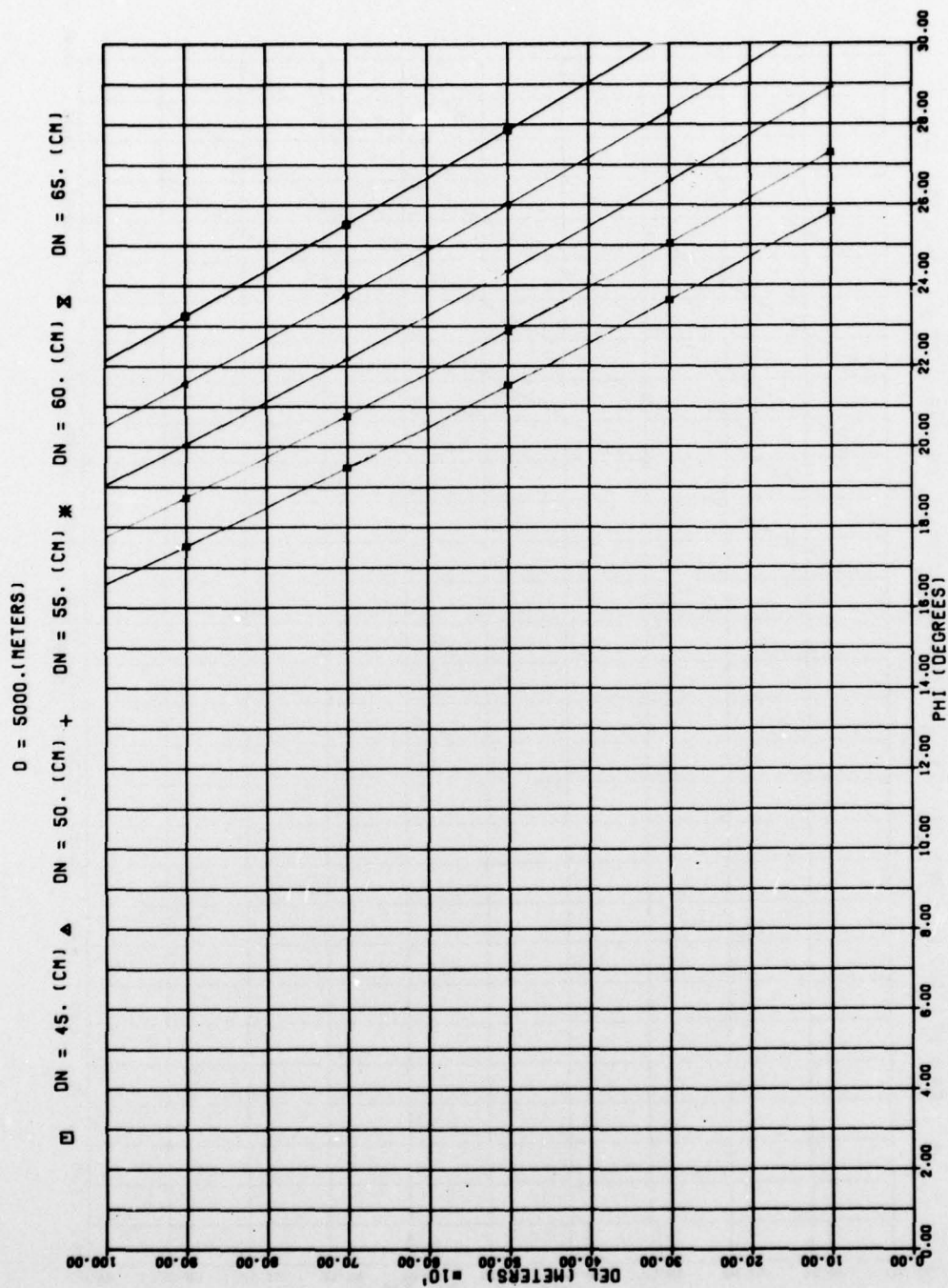






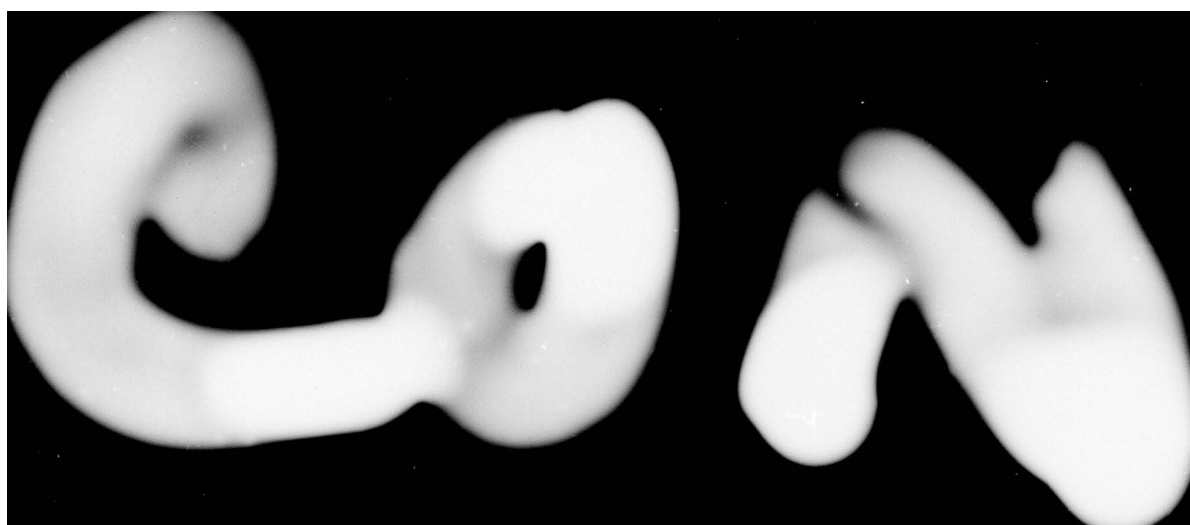








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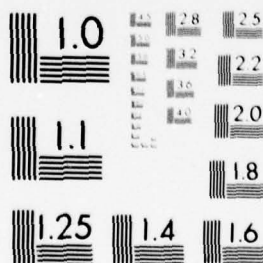
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NAVAL SURFACE WEAPONS CENTER  
DAHLGREN, VIRGINIA 22448

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$$\begin{aligned}
& F_U(\alpha, \beta) \\
&= 2\pi \left\{ -\frac{\alpha^2 \beta^2}{2(1+\alpha^2)} + \frac{1+\alpha^2(1+\beta)}{2\alpha(1+\alpha^2)} \sqrt{1+\alpha^2(1+\beta)^2} \right. \\
&\quad + \frac{\alpha \beta^2}{2(1+\alpha^2)^{3/2}} \ln \left[ \frac{\alpha \beta (\sqrt{1+\alpha^2} - \alpha)}{\sqrt{1+\alpha^2} \sqrt{1+\alpha^2(1+\beta)^2} - 1 - \alpha^2(1+\beta)} \right] \\
&\quad \left. - \beta - \frac{1}{2} \right\} \quad \text{for } \beta \geq 0 \quad (5)
\end{aligned}$$

$$\begin{aligned}
& F_I(\alpha, \beta) \\
&= 2\pi \left\{ \frac{\alpha^2 \beta^2}{2(1+\alpha^2)} + \frac{1-\alpha^2(\beta-1)}{2\alpha(1+\alpha^2)} \sqrt{1+\alpha^2(\beta-1)^2} \right. \\
&\quad + \frac{\alpha \beta^2}{2(1+\alpha^2)^{3/2}} \ln \left[ \frac{\alpha \beta (\sqrt{1+\alpha^2} + \alpha)}{\sqrt{1+\alpha^2} \sqrt{1+\alpha^2(\beta-1)^2} - 1 + \alpha^2(\beta-1)} \right] \\
&\quad \left. - \beta + \frac{1}{2} \right\} \quad \text{for } \beta \geq 1 \quad (6)
\end{aligned}$$

No sphericity corrections have been made in the calculation of the geoidal elevations.

The limiting value of  $F_U$  when  $\beta \rightarrow 0+$  is  $\pi \left\{ \frac{\sqrt{1+\alpha^2}}{\alpha} - 1 \right\}$ . This corresponds to a vanishingly small depth of the peak.